LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

THIRD SEMESTER – **NOVEMBER 2023**

UMT 3502 – DIFFERENTIAL EQUATIONS AND LAPLACE TRANSFORM

Date: 04-11-2023 Dept. No. Max. : 100 Marks

Time: 09:00 AM - 12:00 NOON

	SECTION A - K1 (CO1)	
	Answer ALL the Questions	**(10 x 1 = 10)**
1.	**Answer the following**	
a)	Eliminate a and b from $xy = ae^x + be^{-x}$	
b)	Solve $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 4y = 0$.	
c)	Eliminate the arbitrary function from $z = f(x^2 + y^2)$	
d)	Evaluate $L(t^2 + 2t + 3)$.	
e)	Evaluate $L^{-1}(\frac{s}{s^2+9})$	
2.	**Fill in the blanks**	
a)	A differential equation is an equation in which_____ occur.	
b)	The solution of the equation $a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = 0$ is called the _____.	
c)	The solution of the equation $f(p, q) = 0$ is of the form _____.	
d)	$L(e^{-st}) =$_____	
e)	$L^{-1}(sinat) =$_____.	
	SECTION A - K2 (CO1)	
	Answer ALL the Questions	**(10 x 1 = 10)**
3.	**Chose the Correct Answer**	
a)	The order and degree of the differential equation $2\frac{d^2y}{dx^2} + (\frac{dy}{dx})^2$ is i) 1,2 ii) 2,1 iii) 2,2 iv) none of the above	
b)	The roots of the auxillary equation of the differential equation $\frac{d^2y}{dx^2} - 4y = 0$ i) 2,2 ii) 2, -2 iii) -2,-2 iv) none of the above	
c)	A solution containing as many arbitrary constants as there are independent variables is called i)complete integral ii)particular integral iii) single integral iv) none of the above	
d)	$L(t^n) =$ i) $\frac{n!}{s^{n+1}}$ ii) $\frac{n!}{s^n}$ iii) $\frac{n}{s^{n+1}}$ iv) none of the above	
e)	$L^{-1}(e^{at}) =$ i) $\frac{1}{s+a}$ ii) $\frac{1}{s-a}$ iii) $\frac{1}{s}$ iv) none of the above	
4.	**True or False**	
a)	The order of an ordinary differential equation is of the order of the highest derivative occurring in it.	
b)	The complementary function and the general solution are different for $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 5y = 0$.	
c)	A solution of a partial differential equation is a relation between the dependent and the independent variables that satisfies the differential equation.	

d)	It is a necessary condition that the function should be of exponential order to have Laplace Transform.
e)	The Laplace Transform can be used to solve a system of differential equations.
SECTION B - K3 (CO2)	
Answer any TWO of the following	**(2 x 10 = 20)**
5.	Solve $\frac{dy}{dx} + y\,cosx = \frac{1}{2}sin2x.$
6.	Solve $(D^2 - 3D + 2)y = sin3x$
7.	Solve $\frac{d^2z}{dx^2} = a^2z$ given that when $x = 0, \frac{dz}{dx} = a\,siny$ and $\frac{dz}{dy} = 0.$
8.	Obtain a complete integral of $xp^2 - ypq + y^3q - y^2z = 0.$
SECTION C – K4 (CO3)	
Answer any TWO of the following	**(2 x 10 = 20)**
9.	Solve $\frac{dy}{dx} - y\,tanx = \frac{sinx\,cos^2x}{y^2}$
10.	Evaluate (i) $L\left(\frac{1-e^t}{t}\right)$ (ii) $\int_0^\infty \frac{e^{-t}-e^{-2t}}{t}dt$
11.	Find $L^{-1}(\frac{s-3}{s^2+4s+13})$
12.	Solve the equation $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} - 3y = sint$ given that $y = \frac{dy}{dt} = 0$ when $t = 0.$
SECTION D – K5 (CO4)	
Answer any ONE of the following	**(1 x 20 = 20)**
13.	(i) Solve $xp^2 - 2yp + x = 0.$ (10 Marks) (ii) A boat is rowed with a velocity u directly across a stream of width a, if the velocity of the current is directly proportional to the product of the distances from the two banks, find the path of the boat and the distance downstream to the point where it lands. (10 Marks)
14.	Solve $\frac{d^2y}{dx^2} + y = secx$ using the variation of parameter.
SECTION E – K6 (CO5)	
Answer any ONE of the following	**(1 x 20 = 20)**
15.	(i) Find the general solution of $(y + z)p + (z + x)q = x + y.$ (ii) Solve $p(1 + q^2) = q(z - 1).$ (10+10 Marks)
16.	Solve the simultaneous equations $3\frac{dx}{dt} + \frac{dy}{dt} + 2x = 1$ and $\frac{dx}{dt} + 4\frac{dy}{dt} + 3y = 0$ given that $x = 0 = y$ at $t = 0$ by using Laplace transform.

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